THE LABORATORY METHOD IN THE SECONDARY SCHOOL.[1]

THE last few years have wrought great changes in teaching in general, and in mathematical teaching in particular. Very few of us would refuse to admit that most of these changes are in the direction of marked improvement. The number who regard mathematical teaching incapable of improving is diminishing year by year. The changes referred to are both the cause and the effect in large degree, of the extensive establishing of normal schools and of departments of education in connection with the leading colleges and universities, and to the consequent awakening of the general public to an active interest and participation in the administration of school systems and of school work. American teachers are the beneficiaries of the stimulus of an enlightened public opinion on school matters to an extent hitherto unheard of in any other age or country. As a result there are more good teachers and good teaching today than the most sanguine student of education of ten years ago would have dreamed of. The progress of the last ten years has been without a parallel in our educational history, and "forward and upward to higher and better things" are the watchwords of a host of mathematical teachers.

It has been longer than twenty years since the faculties of American universities and colleges generally began to conceive their function in American civilization to be threefold; viz.: to conserve, to disseminate, and to extend the world's present store of knowledge. It has been hardly ten years since universities became fully conscious of practical consequences springing from the last two phases of their duty. Because it was cheaper, easier, more dignified, and required less will power to perform the offices of conservator than those of the disseminator and

[1] Read before the Mathematical Club of the University of Chicago, August 21, 1903; at the close of the address a number of home-made mathematical apparatus were exhibited and explained.

investigator, the former until very recently were magnified in all but a very few institutions almost to the total eclipse of the latter. As a result, most of our university and college faculties even today comprise many enthusiastic curators, comparatively few progressive teachers, and but very few energetic investigators. Consequently, in the mental make-up of the typical mathematical teacher of today the elements of conservatism, of pedagogical progressiveness, and of research enthusiasm enter about in proportion to the number of members of university faculties representing the three phases of the university function.

But the extensive establishment of normal schools and of university departments of education among us is rapidly shifting the center of gravity of attention of both the educational clergy and laiety to the last two offices. Teachers—mathematics teachers—even now show a general tendency to define the boy as the "young man," the immature man, and not as the "little man," as was their wont until recently. They have already learned the pedagogical unsoundness of the sentiment;

"Men are only boys, grown tall,
Hearts don't change much after all,"

as applied to the heads and have even cast grave suspicion upon it as applied to the "hearts." They have proved that its truth would be enhanced and its poetical effect not greatly reduced by changing it to run:

Men are not mere boys, grown tall,
Heads do change much after all.

Speaking more prosaically, it has been conclusively demonstrated that the teacher cannot prescribe successfully for the boy by first ascertaining the proper dose for the adult and then reducing it. For the immature patient due regard must be given to quality as well as to quantity. This change in men's views, which is both the cause and the effect of the establishment of the institutions mentioned above, epitomizes the changes now making in mathematical education. The consequences of these changes are the really significant matters for modern education, notwithstanding the unfortunate fact that the small place given to their study and discussion on the programs of the National Educa-

tional Association and of other educational bodies might lead the unwary to believe them to be of little moment compared with matters relating to school administration, which usually usurp the lion's share of the time and energy of these meetings. It is, however, gratifying to note that the little time that is grudgingly assigned to these more consequential matters is sufficient to show an increasing liberality and progressiveness among practical teachers, which some of the leading exponents of educational administration would do well to emulate.

The general open-mindedness to progress and disposition to facilitate it have perhaps been seen at their best among science teachers and scientists. Untrammeled by traditional ideals and unhampered by methods of instruction inherited from an outgrown past, when science courses were introduced into school curricula a few years ago they had before them, in very large measure, a clean page on which to begin the new chapter they were destined to write in the history of nineteenth-century educational method. As a consequence, the closing quarter of that century beheld such a shifting and sifting of educational ideals and practices, such a complete realignment of educational forces, as has hardly been witnessed before in the history of human thought. The old skins of methodology were no longer able to contain the new wine of the modern subjects, and there rapidly sprung up in education what soon came to be known as the *scientific method*, or the *method of science*. The virtue of the new method soon came to be recognized generally by leading teachers in other than scientific fields. We have, indeed, grown familiar with the phrases, the scientific method of Latin, of Greek, of grammar, of history, of law, of economics, of pedagogy, of music, of morality, and even of art. Indeed, the range of uses and of abuses of the phrase "scientific method" has become so great as to render well-nigh hopeless the attempt to define its content with all embracing adequacy.

But whatever the phrase may be understood to mean in the multifarious connections in which it does service nowadays on the title-pages of text-books, when applied to the sciences the "scientific method" connotes at least two important notions.

Induction as a thought-process and laboratory instruction as an external agency in founding and facilitating this process are perhaps the dominant ideas of the method in question. It is the contention of the present paper that much fuller conformity to these two notions in mathematical teaching than has yet been attempted would result in the greatest good in secondary teaching. The particular purpose of this article is to gather together some of the main reasons for the extensive use of the laboratory feature with high-school pupils and to give a few general indications as to the way to use this feature with high-school classes.

It is trite in theory, though not in practice, that the prime requisite of any method of teaching immature pupils should be psychological, rather than logical. But even at the risk of doing violence to this well-known principle at the outset of my discussion, it seems well to begin by giving some definiteness to the idea under discussion. Let us then consider for a moment the oft-repeated query of the mathematical teacher: "What is the laboratory method anyhow?"

As suggested above, to undertake to define adequately any notion which, like the one in question has gained professional currency in both original and figurative senses, in such a way as to be generally acceptable to those who use it, would be as unwise as it is useless. This is particularly true of a notion with which common usage has fully familiarized us and of which we have a better working understanding before than after defining it. Furthermore, with an idea such as is contained in the phrase "laboratory method," which in its administration is capable of a wide latitude of legitimate interpretation and variation, precise definition is not only passively futile, but may even be positively harmful. To define too precisely an idea none too precise in itself invites the shafts of the constitutional critic and helps him get his target in range. Just now mathematical teaching needs something more positively constructive than either criticism or conservatism. Finally, the essence of the idea is obvious. Fundamentally, laboratory method means *work*—work *on the pupil's part*—or, better still, *method of getting work done by the pupil on his own initiative, under the impulse of his natural interests,*

and largely under the guidance of his own intelligence. The laboratory method merely condenses to its essence the old, but ever new pedagogic maxim : "Learn by doing, and do while learning."

In mathematics, the method of the laboratory calls to the traditional formalist an abrupt, "About, face!" It commands him to face knowledge and knowledge-getting from the standpoint of the learner, rather than from that of the master. On so broad and solid a pedagogical platform as this, certainly every live teacher—every teacher with a forward rather than a backward yearning—can find a place to stand. Indeed, one of the strongest arguments for the introduction of the laboratory idea in mathematics teaching is its flexibility—its capability of adaptation to the requirements of so wide a range and variety of conditions and situations.

Granting at once that mathematics is essentially abstract, that in the measure in which the pupil is bound to the concrete he is not a mathematician at all, it must nevertheless be admitted by all who have had much contact with high-school classes that the thought processes of the secondary pupil become mathematical very slowly. Mathematical method is the method of the adult to a degree not generally recognized in actual teaching. The secondary pupil is only beginning to become reflective, and even so far as his habits of thought are reflective, they are very far from analytically reflective. His normal interest is chiefly concerned with vague *subjective* wholes, much as the early elementary pupil finds his chief interest in vague *objective* wholes. Here as elsewhere interest is an index of capability. There is therefore but feeble native power in the early secondary pupil to deal profitably with the analytic and synthetic products of reflection. To undertake to force the exercise of mathematical reflection prematurely is in its first stages distasteful to the pupil, and finally disgusting and stultifying to him. This is just what our early foisting of abstract mathematics upon him too often does for him in practice. Talk as learnedly as we please to him, he still persists to live in the world of things. During the first two years of the high school the student is able to seize thought-relations only by a very slender thread of analogy to

thing-relations. If his experiential knowledge of things is meager, as is more often the case than most teachers think, his foothold is well-nigh as uncertain as his footsteps, and under such circumstances, he can never hope to lay vigorous hold on mathematical truth and method. The laboratory method, in the language of the scientists—who are best qualified to speak on this subject—means "work with things and ideas about things on the basis of clearly comprehended premises, under the stimulus of a clearly conceived purpose, and under the guidance of a teacher who has been over the road the pupil is traveling." We have long since outgrown the notion, once quite general among teachers, that artificial difficulties are superior to natural difficulties as instrumentalities for seasoning and toughening mental fiber and sinew. An advantage that can hardly be overestimated of the laboratory procedure with mathematical classes is that pupils *sense* the difficulties to be overcome *as real* and *natural*, actually needing to be resolved and demanding a knowledge of the mathematical tool as a means of their resolution. In short, it recognizes the educational importance of letting the student know both *how* and *why* he must use the mathematical tool to get on well in any line of study.

With the secondary pupil, to be continually baffled, or too frequently defeated, is discouraging to the point of disheartening. The secondary pupil is inordinately sensitive to failure. With him success is emphatically the mother of achievement. The successes need not be great, but they must not be too long postponed. It is too much to expect him to keep up his interest in mathematical study merely in the hope of some future good, dimly perceived by him if at all, in the face of uninterrupted failure. It is only the mendicant, or the moral weakling, who is satisfied to remain the subject of alms-giving. The laboratory in mathematics derives no inconsiderable part of its merit from the circumstance that it furnishes a goodly crop of those minor successes along the way that are so indispensable to the secondary pupil in keeping up the tension between effort and ideal by giving him more than the occasional feeling that if he is not yet independent he is at least gaining in independence.

A view formerly held quite widely by high-school superintendents and mathematical supervisors, and which, though still extant, is rapidly yielding to rational thought, is that a multitude of pedagogical sins are covered in a mathematics teacher of whom it may be said "he gets the work done." Usually also this phrase means that an assigned number of problems, topics, or pages are covered. We have at length come to see the truth — though we do not yet realize to the full the consequences of our seeing — that school work done under the stimulus of the ferule and rattan, even though these ancient pedagogical insignia give place to the dread of ever-impending failure or to the more modern insignia of the acid face and astringent tongue of a teacher — school work done under the mere task-master is now understood to be of the lowest value of all. The *way of getting school work done* is of much more consequence than is the mere doing of it. Is the work purposeful? and, Are the workman's acts conspicuously purposive to both teacher and pupil? are the all-important questions for education. All other work is drudgery and degrades both teacher and pupil. The motiveless mathematics teacher is always resorting to mechanical drill. The real teacher gets better results with little or no formal drill, by filling all his work, drill included, with purposeful interest. Not work, but the right kind of work, educates in the mathematical class-room as everywhere, and this is what the intelligently conducted laboratory secures.

We mathematical teachers have become noted, not to say notorious, for our persistent and strenuous insistence upon the high value of mathematical study, as a discipline in clear thinking. But we need to remember that here again it is not thinking, even though it be clear, but the right kind of clear thinking, that educates. Clear thinking about the number of $5 bills needed to cover a town lot of given dimensions is of no great educational value. Thinking that does not eventuate in right doing is also of inferior educative value. In the last analysis it is the strong will, the healthy conscience, and the facile hand quite as much as the clear head that count in realizing the ends of education. This may be pedagogical commonplace in many

school subjects, but it is not so in mathematics. The most telling weakness of current mathematics teaching is that it makes so feeble an appeal to the will. Most mathematical subject-matter is morally colorless, and instruction which addresses itself almost exclusively to the intellect affects character but feebly at best, and when, as is too often the case with much mathematical instruction, the memory alone is appealed to, the result is a searing over rather than a quickening of the mental faculties. What is most needed in mathematics in the schools today, and particularly in the secondary school, is a subject-matter which makes an all-around appeal to the student, which, without neglecting the perceptive and rational faculties, will reach the will and result in strengthening the executive faculties. The laboratory in secondary mathematics calls for right-doing as well as right-thinking. It materializes the quantitative study of things and relations about things, calls for the *acts*, not the results merely, of perception, analysis, synthesis, inference, and generalization, thereby reducing to the habitual those particular mental processes which characterize the mathematical investigator. In this work the whole individual must engage, senses, hand, and head being enlisted vigorously and continually.

But, says someone, consideration of economy of time and energy on the pupil's part make it advisable for the mathematical master to abstract the subject-matter from the confusing details of the concrete situations which call for mathematical treatment. In other words, we are reminded that the teacher should do the generalizing and abstracting and assorting of the mathematical ideas, to the end that the learner may concentrate his forces against the mathematics as such. This is strictly equivalent to saying that, since the body needs a certain quantity of albumen, of proteids, of fat, of starch, of oxygen, of nitrogen, and of hydrogen for its proper nourishment, the best dietetics demands that all these substances be taken into the system in their unmixed and elemental purity. But physical food, so taken, is neither so appetizing nor so nutritive as when the food constituents are properly correlated into the forms in which nature indicates they should be taken by placing the required combina-

tions around us within easy access. It is precisely so with the taking of mental food. No amount of isolation of the mental food constituents of mathematical study, with a view to reducing to a minimum the energy of the digestive process which we assume to be wasted in the work of eliminating from the system such irrelevant ingredients as are not needed and cannot be assimilated would be of service. The organs of physical digestion as those of mental digestion have this function to perform. To undertake to carry on the processes of physical nutrition by feeding the body on chemically pure elements would be to attempt to reduce the body to a machine, and, were it possible to do this, it would, of course, permanently paralyze nature's agencies of digestion and nutrition. This would be serious, even if it were certain that a sufficient supply of chemically prepared food constituents were always available. And so our attempts to mechanize the mental processes of separation, assimilation, and nutrition in mathematical teaching result in distaste, nausea, and ultimately in the atrophy of the mathematical faculties. When the pupil leaves his teacher and his text-books, he must take the mathematical problems in the form in which nature and the industries set them before him, and, if he is to deal successfully with them, must be able to discern readily the abstract in the concrete, to disengage from confusing material situations the quantitative elements and relations through which the mind must grasp, systematize, and hold them, if it is to understand and use them. After this analysis and generalization—which are the really difficult phases of a mathematical problem, and with which current mathematical teaching concerns itself almost not at all—it is but a simple matter to apply the mathematical tool and to derive the necessary quantitative consequences. This last is now receiving almost exclusive attention in school work.

Laboratory work with real problems, in the formulation and handling of which the pupil habituates himself to the transition from the concrete to the abstract, trains the faculties of analysis and abstraction, teaches him to make his own mathematical problems, to grow his own mathematics, and goes far toward supplementing the too isolated and too abstract teaching of sec-

ondary mathematics of today. If at the other end of the teaching problem much more attention than is customary be given to the concrete interpreting and construing of the results reached in the abstract part of the work, as this work is now being done, the steps from the abstract back to the concrete again may be made much easier to the pupil. This would go far toward restoring the connections of abstract mathematical work with the world of reality, and would deprive of most of its pertinency and force the charge now quite common against secondary mathematics that it is isolated from everything of which practical people have any need or in which they can conceive an interest.

Practical teachers know that the best students of mathematics are neither those who take the subject because it is required nor those who take it because they love it, but that they are those who think they need it and feel that they must have it. From his natural interest in the larger truths of the subject the lover of mathematics is too often ready to pass lightly over the smaller, though very important, truths thus laying a faulty foundation which will cause both him and his teacher much trouble later. The teacher must keep close watch over him to prevent this. On the other hand, the student who feels he must have the subject is always willing to attend to necessary details, "to calk the joints," for fear that the truth he passes slightingly over be just the one he may need soon, and with reference to which, if his understanding be imperfect, he may be led a little later into serious consequences. If the social, industrial, and school environment be made to contribute largely to the mathematical work, this third class of mathematical student may be very greatly increased, and students may be led, earlier than at present, to see that to get on well in almost any important line of endeavor, a sound knowledge of mathematics is necessary. With this sort of mathematical work there will be less source for regret than is the case at present, among people who have finished their school work, that they did not give more attention to mathematical study when in school. Another salutary effect of this treatment of mathematics is that the element of truth

contained in the widely prevalent saying, "A mathematical teacher is fit for nothing but to teach mathematics," would be lessened through the requirement that the successful prosecution of such mathematical work in the schoolroom would, perforce, have a socializing influence upon him. In this work the mathematical teacher must learn both how to diagnose and to prescribe for social, economic, industrial, scientific, and for many other needs, besides those of the mathematics class-room. It has already been shown, however, that this difficulty is not serious in practice. The mathematics teacher is as anxious as anyone to have his work appreciated, and is willing to qualify if he is not already fitted for such work.

In the writer's opinion laboratory work in mathematics means also a good equipment of material apparatus. The mathematical equipment of the high school should consist of any, or all, the instruments and appliances of the following list that the school board can be induced to buy. Such as are in the physics laboratory and are available for mathematical uses of course need not be duplicated.

A FAIRLY COMPLETE EQUIPMENT FOR A MATHEMATICAL LABORATORY.

1. Set of drawing instruments, drawing board, T-square, and 30°- and 45°-triangles for each pupil.

2. Supply of India inks and of drawing paper; also individual notebooks supplied with cross-ruled paper.

3. A large and well-lighted recitation room fitted with good drawing desks.

4. Carpenters' tapes, surveyors' tapes, and architects' scales.

5. Three-, five-, and seven-place logarithmic tables. The three-place table of numbers and of trigonometric functions on one side of a small card; the five-place tables of numbers and of trigonometric functions printed on one side of large cards—say $16'' \times 24''$; and the seven-place tables printed on large cards and mounted around a post or column, as is done with large photographs in museums. The pupil should be taught to select from these tables for a given computation the three-, five-, or seven-place table according to the measure of accuracy justified by the unavoidably inaccurate data he is using.

6. Logarithmic slide rules and computing machines.

7. A surveyor's compass, a transit, and level, and leveling rods and flag-poles.

8. A surveyor's plane table and a sextant.

9. Weighing apparatus, as steelyards, balances, etc.; pendulums, barometers, and thermometers.

10. Force appliances, such as cords, pulleys, etc., and the simple machines.

11. One hundred good texts on arithmetic, algebra, geometry, trigonometry, physics, elementary mechanics, and astronomy; also Crelle's *Multiplication Table.*

12. A dozen treatises on these subjects, and a few good histories of mathematics and of the mathematical sciences.

13. Spherical blackboards, both concave and convex.

14. Three plane blackboards for projective and descriptive work in geometry.

15. Mathematical models.

16. Samples of actual engineering and architectural office drawings of machines and structures displayed continually before the class.

17. Gyroscope taps.

18. A set of Hanstein's models for projective work.

19. Stereopticon and slides.

The apparatus should be used by the classes in mathematics, not so much to find the physical properties of the apparatus themselves, such as contraction or expansion due to changes of temperature, as to furnish measures to be used in the solution and discussion of all kinds of quantitative problems. Not the properties of the tape, but the properties of linear magnitudes, should receive the attention of the mathematical pupil. In the mathematical laboratory it is not so important that the best modern form of the experiment be adhered to as that the mathematical ideas be fully realized by the pupil.

A LESS PRETENTIOUS, BUT VERY USEFUL, MATHEMATICAL EQUIPMENT.

1. Pencils, right-line pen, compasses, ink and paper, drawing board and cheap triangles for each pupil.

2. One set of good drawing instruments for class use.

3. Individual notebooks supplied with cross-ruled paper.

4. Outfit of home-made surveying instruments (see Myers's pamphlet on *Observational and Experimental Astronomy;* the Ravenswood Press.)

5. A Woodworth sextant (Lewis Institute, Electrical Laboratory; price, \$1).

6. Carpenter's fifty-foot tape, and an architect's scale.

7. A dollar logarithmic slide rule; also Crelle's *Multiplication Table.*

8. Three- and five-place logarithmic tables.

9. Home-made models of the most familiar geometrical forms.

10. Twenty good texts, treatises, and histories of algebra, geometry,

trigonometry, elementary mechanics, and astronomy kept in class-room for reference use.

11. Cheap spherical blackboards.
12. A home-made plane table with alidade (see Myers's pamphlet).

Numerous suggestions as to problems and methods of handling them can be obtained from Appendixes I, II, and III of the *Annual Report of the Central Association of Science and Mathematics Teachers.* These appendixes contain the report of a committee appointed last December to consider the question of the correlation of mathematics and physics in high-school teaching. The committee consisted of ten members—five representing mathematics and five representing physics—all of whom are experienced teachers who have for some time been correlating the subject-matter of mathematics with the allied subjects, and nearly all of whom have had considerable experience in high-school teaching. The report represents an immense deal of good work and sound pedagogical thought on the teaching of secondary mathematics. The report can be had of the association's treasurer, Mr. E. C. Woodruff, Lake View High School, Chicago. It is worth many times the price (15 cents) asked for it.

No better information as to the way to use the mathematical library under the laboratory plan can be given here than to refer those interested to Mrs. A. K. Hornbrooke's admirable little pamphlet on *The Laboratory Method in the High School,* published by the American Book Co., Chicago (price, 10 cents). No skeptical teacher can read this pamphlet carefully without becoming a convert to this method for high-school mathematics classes. It should be laid aside only by such as do not wish to become converts.

A few words may now be permitted as to the course of study for the mathematical work of the secondary school. Here again it seems desirable to become a little visionary, inasmuch as no sequence of subjects nor distribution of time which is consistent with present high-school programs can accomplish the best results from a mathematical point of view. Granting that the interests and purposes of mathematics in the high school must be subordi-

nated to the interests and purposes of high-school education as a whole, it is still worth while occasionally to study programs from the specific points of view of the individual subjects. This sort of study would be justified if its sole purpose were to find to what extent the requirements of the special subjects are being sacrificed to the good of the whole, and whether the circumstances of the situation call for so full a surrender of the interests of the special subjects. The program proposed here assumes the period of secondary education to begin with the seventh grade of the elementary school, and it follows the work only to the college. The course assumes five hours a week continuously to be given to the mathematical work, and the numbers placed beside the separate subjects are intended only as rough indications of the way these five hours should be proportioned in these particular subjects. The science and hand-work which should be carried along from year to year as a basis for much of the mathematics are also indicated.

PROPOSED COURSE IN SECONDARY MATHEMATICS.

Subject	Seventh Grade	Eighth Grade	Ninth Grade	Tenth Grade	Eleventh Grade	Twelfth Grade
Arithmetic	2	1	1	1	..	..
Algebra	1	2	2	2	3	..
Geometry	2	2	2	2	2	2
Trigonometry	..	..	..	..	..	3
Total	5	5	5	5	5	5
Nature study	1	2	..	..	..	..
Elem't'ry phys. and mechanics	..	..	2	2	2	2
Manual training	1	1	2	2	2	..
Drawing and modeling	1	1	..	..	..	..

The geometry of the seventh and eighth grades should be largely observational, experimental, and constructive work. A little demonstrative work would come in incidentally in the eighth grade. Much of the geometry of the second-year high school (ninth grade) should be geometrical drawing. That of the last year (twelfth grade) would be mainly solid geometry. The work of both the last years would be demonstrative geometry. The manual training of the tenth and eleventh grades should be

pretty largely taken up with the devising and constructing of apparatus for the physical and mathematical laboratories. The seriousness of the customary break in mathematics in the third year of the high school is difficult to overestimate and, the writer fears, is usually underestimated. Connection and continuity are everything in mathematical study, and mathematical power in the pupil can be acquired only through slow, gradual, and uninterrupted growth. Mathematical knowledge and power are emphatically functions of the time. It is confidently believed that the best interests of sound secondary education call for a much wider recognition of these important truths than is customary with supervising officers.

In conclusion it may be said that the laboratory method of teaching mathematics among other things means the correlation of the subject-matter of mathematics and the metrical sciences into an organic unity of truth whose educative value is to be read in the broadening and deepening life of the pupil, and not at all in the perfection of the subject-matter studied. It means that the teachers of mathematics and of science shall get together in planning their work, each taking into cognizance the work of the other, each striving to supplement and reinforce the work of the other. They must recognize, not only that they have the same pupils, but also that the fundamental concern of both should be, not how much mathematics or how much physics, but, rather, "What is it to educate young men and women, and how may I increase my efficiency in the discharge of my duties as a teacher of youth?" These are momentous questions, and to answer them from the standpoint of the mathematical teacher is the supreme mathematical problem of the age.

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